

Maximal CP and Bounds on the Neutron Electric Dipole Moment from P and CP Breaking

Ravi Kuchimanchi*

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We find in theories with spontaneous P and CP violation that symmetries needed to set the tree level strong CP phase to zero can also set all non-zero tree level CP violating phases to the maximal value $\pi/2$ in the symmetry basis simultaneously explaining the smallness of $\bar{\theta}$ and the largeness of the CKM CP violating phase. In these models we find the one loop lower bound $\bar{\theta} \geq 10^{-11}$ relevant for early discovery of neutron EDM $d_n \geq 10^{-27}$ ecm. The lower bound relaxes to $\bar{\theta} \geq 10^{-13}$ or $d_n \geq 10^{-29}$ ecm for the case where the CP phases are non-maximal. Interestingly the spontaneous CP phase appears in the quark sector, not the Higgs sector, and is enabled by a heavy left-right symmetric vectorlike quark family with mass M . These results do not vanish in the decoupling limit of $M_{H_2^+} > M \rightarrow \infty$ (where $M_{H_2^+}$ is the mass of heavy Higgses at the parity breaking scale) and the age-old expectation that laws of nature (or its Lagrangian) are parity and matter-antimatter symmetric may be testable by the above predictions and EDM experiments, even if new physics occurs only at see-saw, GUT or Planck scales. There is also a region in parameter space with $M_{H_2^+} < M$ where the above bounds are dampened by the factor $(M_{H_2^+}/M)^2$. By using flavour symmetries and texture arguments we also make predictions for the CKM phase that arises from the maximal phase on diagonalization to the physical basis. There are no axions predicted in this model.

I. INTRODUCTION

With the discovery of Higgs or Higgs-like boson the only standard model parameter that remains to be determined is the value of the strong CP phase $\bar{\theta}$ that requires the violation of both left-right (or parity P) and matter-antimatter (CP) symmetries. Over the past 6 decades searches for the neutron electric dipole moment (EDM) which is also both P and CP odd have provided the experimental bound $d_n \leq 1.9 \times 10^{-26}$ ecm at 90% C.L. [1]. With $d_n \sim 2 \times 10^{-16} \bar{\theta}$ [2] this translates to $\bar{\theta} \leq 10^{-10}$. Several efforts [3] are currently underway to improve the sensitivity of these experiments by two orders of magnitude that can potentially probe d_n down to 10^{-28} ecm or $\bar{\theta} \sim 10^{-12}$. Storage ring experiments [4] being planned to search for EDMs of the proton and deuteron can further this frontier to an equivalent of $d_n \sim 10^{-29}$ ecm or $\bar{\theta} \sim 10^{-13}$.

Since both P and CP are broken in nature we would naively expect $\bar{\theta} \sim \gamma$ where γ is the CP violating CKM phase. However experimentally $\bar{\theta} \ll \gamma \sim 69^\circ$ which hints at a hidden symmetry, and this large inequality is the well-known strong CP puzzle.

The most popular solution to the strong CP problem is the Peccei-Quinn (PQ) symmetry [5] that dynamically sets $\bar{\theta}$ to zero. The neutron EDM induced radiatively in the standard model in this case is $d_n \sim 10^{-32}$ ecm and is too small to be observed in the ongoing experiments. There can be beyond the standard model contributions to d_n due to new physics such as supersymmetry but these decrease quadratically as the scale of new physics becomes large. For example with supersymmetry as new

physics, $d_n \sim 10^{-25} (TeV/M_{SUSY})^2$ ecm [2] and becomes smaller than the sensitivity of ongoing experiments for $M_{SUSY} > (10 - 100) TeV$.

In this work instead of using the PQ symmetry we follow a different line of approach to solve the strong CP problem and show that the model presented in this paper (along with that in reference [6]) can lead to a sizable d_n that is discoverable in the ongoing EDM experiments *even if all new physics occurs only at very high scales such as GUT or Planck scales*.

Since violation of both P and CP is fundamental to the existence of a non-zero d_n the approach different from the continuous PQ symmetry is to impose either of these discrete symmetries to set $\bar{\theta}$ to zero [6–8]. The challenge then is that P/CP must break without spontaneously generating a strong CP phase at the tree-level so that the experimental constraint $\bar{\theta} \ll \gamma$ is respected. In the past this has typically required along with vectorlike quarks further symmetries that generate the Nelson-Barr form of mass matrices as in [7] or mirror parity defined so that it takes known quarks and leptons to 3 additional generations of mirror quark and lepton families as in [8].

However more recently in [6] we showed that with no other symmetries imposed and no multiplication of existing families by three generations of mirror families, just P and CP are sufficient to solve the strong CP problem in the left-right symmetric model with the addition of a full vectorlike quark family. A crucial aspect of this model is that terms and vacuum expectation values (VEVs) that violate P conserve CP and those that violate CP conserve P and therefore the strong CP phase is not generated at the tree level as it is protected by either P or CP for every term. Our work in [6] is the first solution of the strong CP problem in the left-right symmetric model where P (and not PQ symmetry) is used to set the tree-level $\bar{\theta}$ to zero. (See also [9].)

* raviaravinda@gmail.com

Moreover $\bar{\theta}$ generated radiatively at the one-loop level in this model does not diminish as the scale of P and CP breaking (that is mass scale $M_{H_2^+}$ of Higgs sector that breaks P and the mass M of vectorlike quarks that are needed to break CP) go to infinity, since CP phases generated in collusion with Yukawa terms do not respect the decoupling theorem. d_n generated in this model can be much greater than the naive standard model expectation of $10^{-32}ecm$ even if there is no new physics at TeV or 1000's of TeV scales and therefore should be of interest to the ongoing neutron EDM experiments.

We first generalize the model of reference [6] to allow for spontaneous CP violation (instead of softly through dimension 3 fermion mass terms as in that work) so that we can consider both the case of spontaneous (which could also be more predictive) as well as soft CP breaking.

To our surprise we find that thus solving the strong CP problem not only determines the strong CP phase, but also determines the CP phases in the quark mass matrices to be maximal (ie) $\pi/2$ in the symmetry basis, consistent with the high value of the CKM CP violating phase $\gamma \sim 68^\circ$ obtained on diagonalizing to the physical mass basis. While several works such as [10–12] studying the texture of the quark mass matrices have in the past suggested that CP violation could be maximal (either in the mass matrix or in the Jarlskog invariant), the phase $\pi/2$ is usually put in by hand and not obtained by symmetries.

Spontaneous breaking of CP is an attractive idea as it makes CP phases calculable. In practice however the phases are determined by minimizing the Higgs potential that has several additional parameters due to which we lose predictivity. What we show is that once we have both P and CP symmetries imposed and solve the strong CP problem, the phase of a CP violating VEV is determined by its P transformation properties independent of the Higgs parameters and is maximal in the quark mass matrix (that also has vectorlike quarks) in the symmetry basis. See sections II, III A, III B and III C.

It turns out that presence of a maximal CP phase may be experimentally verifiable by the neutron EDM searches. The lower bound on the strong CP phase generated depends on rotations needed to go from the symmetry basis to the physical mass basis. If the CP phase generated in the symmetry basis is $\pi/2$ (as opposed to an arbitrary number that could be chosen to be the observed CKM phase γ for the purposes of calculating a lower bound) then some amount of additional rotation that mixes the real and purely imaginary terms of the mass matrices is needed to obtain the observed $\gamma \sim 68^\circ$. In a large region of parameter space, this generates a higher lower bound $\bar{\theta} \geq 10^{-11}$ (or $d_n \geq 10^{-27}ecm$) and the lower bound would be a couple of orders of magnitude less ($\bar{\theta} \geq 10^{-13}$ or $d_n \geq 10^{-29}ecm$) had the CP phase been arbitrary and not maximal.

The idea of the strong CP problem hinting at a hidden P and CP symmetry that are spontaneously or softly

broken as presented in this work (and in [6]) is verifiable by finding d_n greater than either of the above two lower bounds for the case where the heavy higgses are heavier than the vector like quark masses (ie) $M_{H_2^+} > M$. While there is a suppression factor of $(M_{H_2^+}/M)^2$ if $M_{H_2^+} < M$ and the lower bounds are then reduced and depend on this factor as well. See sections IV A and IV B

In section V we show how texture considerations and flavour symmetries can be used to make predictions for the CKM phase that is generated from the maximal phase on diagonalization from the symmetry basis to the physical basis.

Finally in sections VI and VII we present some comments and concluding remarks.

II. P AND CP PROPERTIES OF $\bar{\theta} = 0$ VACUUM MAXIMAL CP VIOLATION

We assume P and CP are good symmetries that are both broken by VEVs of a set of Higgs fields. In general the vacuum will be made of several Higgs fields with VEVs that conserve at the tree level:

1. both P and CP
2. CP (but not P)
3. P (but not CP)
4. neither P nor CP .

Since $\bar{\theta}$ is both P and CP odd we would expect a strong CP phase to be generated at the tree level by VEVs that conserve neither P nor CP . Now if there is a solution to the strong CP problem that sets the tree level strong CP phase to zero, we would expect that there are no VEVs of the fourth category. That this is in fact the case is *proved* in the next section for a class of models by examining the minimum of the Higgs potential. This means in a class of models, we can visualize the strong CP solving vacuum to be made of several states, with each state being either P even or CP even or both. This property of the vacuum can help determine the phase of CP violation that is generated at the tree level as we now show.

Consider the neutral component ϕ^o of a Higgs fields that picks up a VEV v . Let us say under CP $\phi^o \rightarrow \phi^{o*}$ so that any non-real v breaks CP . Likewise we choose the CP properties of quarks and leptons so that all coupling constants including the Yukawa coupling are real due to CP .

However as discussed, v conserves P if it violates CP so as not to generate a tree-level strong CP phase. Under P if ϕ^o transforms non-trivially, then P conservation is a non-trivial condition that v must satisfy, and its phase gets determined.

Under P we can choose the quarks to transform as $q_{iL} \leftrightarrow q_{iR}$. The Yukawa terms are of the form

$\bar{h}_{ij}\bar{q}_{iL}\phi^o q_{R,j} + hc$ with \bar{h}_{ij} real due to CP. Under P if we choose $\phi^o \rightarrow e^{i\beta}\phi^{o*}$ then P invariance of the Yukawa term together with real \bar{h}_{ij} implies that $e^{i\beta}$ is real. Therefore $\beta = 0$ or $\beta = \pi$ for ϕ^o that have Yukawa couplings with the quarks.

Moreover from the P transformation $\phi^o \rightarrow e^{i\beta}\phi^{o*}$ it is easy to see that $\langle\phi^o\rangle = |v|e^{i\beta/2}$ will conserve P , as is needed for VEVs that violate CP.

Thus the phase of the VEV gets determined in terms of the transformation properties of the field under parity. For VEVs that have Yukawa coupling with the quarks $\beta = 0$ or π and so the phase of $\langle\phi^o\rangle$ which is $\beta/2$ becomes 0 or $\pi/2$. $\beta = 0$ corresponds to the case where both P and CP are conserved by the VEV and $\beta = \pi/2$ violates CP maximally while conserving P . Both types of fields will be present.

There are a few points we make before we can say that the CP violation is also maximal in the quark mass matrix in symmetry basis:

- The phase generated in the quark mass matrix can be rotated away if there are only the usual 3 families. So there is no CKM phase generated unless the quark content is extended. This is resolved by having a full vectorlike quark family in the model as proposed in reference [6]
- Since there are Higgs fields with both purely imaginary ($\beta = \pi/2$) and real ($\beta = 0$) VEVs, if they contribute to the same element of the quark mass matrix then the phase generated in the mass matrix will not be $\pi/2$ owing to their mixing. We will see in the next section that this does not happen due to the same symmetries (we call these Strong CP Solution Symmetries or SCP for short) that are imposed to help solve the strong CP problem. So each term in the mass matrix is purely real or purely imaginary. Thus a maximal CP phase $\pi/2$ gets generated in the quark mass matrix in the symmetry basis. On diagonalization to the physical basis the real and imaginary parts mix providing a CKM phase consistent with the observations.
- CP can also be broken spontaneously using a CP odd, P even real singlet. This case was already discussed in reference [6] and its physics is equivalent to CP being violated softly by dimension 3 mass terms (and with no singlet) as discussed in that work. The attractive feature of this case (with or without the real singlet) is that no other symmetries (ie SCPs) other than parity and CP need to be introduced to solve the strong CP problem. The flip side is that it is therefore possible to generate an arbitrary (ie non-maximal) CP phase in the mass matrix.

We will estimate lower bounds for $\bar{\theta}$ and the neutron EDM that is radiatively generated in the quark mass matrix for both the above cases of CP violation....where it is maximal in the quark mass matrix in symmetry basis

(see section IV A) as well as where it could take on any value (see IV B).

In the next section we set up the P and CP symmetric strong CP solving model and show that CP violation is maximal in the quark mass matrix. Before we proceed a note on the conventions used which are slightly different from those in this section. So far we defined CP transformations so that real VEVs conserve CP . However in left-right symmetric theories the convention is to define P and CP such that real VEVs of doublet ϕ conserve P while they could violate CP . We revert to this standard convention of left-right symmetric theories in what follows, however since the result of maximal CP violation are convention independent the particular convention chosen does not make a difference to the physics.

III. LEFT-RIGHT MATTER-ANTIMATTER SYMMETRIC STRONG CP MODEL

$$SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times P \times CP \times SCP$$

We begin with the left-right symmetric model [13] based on $G_{LR} \equiv SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times P$ with strong CP solving particle content discussed in reference [6] where CP was imposed on dimension 4 terms and broken softly by dimension 3 mass terms, and increase the symmetry to include CP as a good symmetry of the Lagrangian. Therefore a second Higgs bidoublet is added so that it can receive a CP breaking VEV.

The Higgs fields thus consists of the usual $SU(2)_R$ triplet, $SU(2)_L$ singlet Δ_R (and parity related Δ_L) and two bi-doublets ϕ_s and ϕ_a that transform as $(1, 2, 2, 0)$ under G_{LR} . The Higgs bi-doublets and triplets are represented as complex 2×2 matrices (with the triplet matrices being traceless) in the relevant iso-spaces, as is usual in left-right symmetric model.

The quark content of the model is the same as in [6] where in addition to the usual 3 light chiral families we have a vectorlike $SU(2)_L$ doublet family Q_{4L} and Q'_{4R} that transforms as $(3, 2, 1, 1/3)$, and a parity related vectorlike $SU(2)_L$ singlet ($SU(2)_R$ doublet) family Q_{4R} and Q'_{4L} that transforms as $(3, 1, 2, 1/3)$. The prime on the quarks is being used in our notation to clearly label the mirror component of the vectorlike family.

Thus there are two heavy top and two heavy bottom quarks in the model, one pair from the vectorlike doublet and the other from the vectorlike singlet family. Note that instead of $\Delta_{L,R}$, $SU(2)_{L,R}$ doublets $\chi_{L,R}$ can also be used to break parity and the results in this work will apply to them as well. There are the usual 3 generations of leptons and vectorlike lepton families need or need not exist. We do not explicitly mention the lepton sector in this work, other than briefly in the second comment of section VI.

A. P and CP transformations

Under Parity $SU(2)_L \leftrightarrow SU(2)_R$, $\phi_{a,s} \rightarrow \phi_{a,s}^\dagger$, $Q_{iL} \leftrightarrow Q_{iR}$, $Q'_{iL} \leftrightarrow Q'_{iR}$, $\Delta_L \leftrightarrow \Delta_R$. Note that with these assignments real VEV's for neutral (i.e. diagonal) components of bidoublets $\phi_{a,s}$ will not contribute to P violation. $Q_{iL,R}$ with $i = 1$ to 4 correspond to the usual 3 generations of quarks and the normal chiral components $Q_{4L,R}$ of the vectorlike quarks.

We now choose the CP transformation such that a real VEV for ϕ_a will break CP . Under CP we require $\phi_a \rightarrow -\phi_a^*$, $\phi_s \rightarrow \phi_s^*$, $Q_{iL,R} \rightarrow CQ_{iL,R}^*$, $Q'_{L,R} \rightarrow CQ'_{L,R}^*$, $\Delta_{L,R} \rightarrow \Delta_{L,R}^*$.

With these transformations note that CP invariance of the Lagrangian implies coefficients of all terms (including the Yukawa potential) with an odd (even) number of ϕ_a will be purely imaginary (real). Additionally, the Yukawa matrices must be Hermitian (due to P invariance), dimension 3 quark mass matrix with direct mass terms involving vector-like quarks must be real symmetric (due to CP and P), and dimension 2 Higgs mass parameters must be real (due to P).

We will now discuss a further symmetry that needs to be imposed to solve the strong CP problem and see how this also leads to a maximal CP phase in the quark mass matrix in the symmetry basis. For this we discuss the Higgs potential and show that symmetries can be broken through real VEVs in the model.

B. Z_2 Symmetry, Higgs Potential and P Conserving, CP Violating VEV

If all parameters of the Higgs potential are real then all VEVs that minimize the Higgs potential can be naturally real. Note that a real VEV for ϕ_a will break CP and not P and is of interest to us based on arguments of section II.

However there are P and CP invariant quartic terms with an odd number of ϕ_a that have purely imaginary couplings and are dangerous for example,

$$iTr(\lambda' \phi_a^\dagger \phi_s + \lambda'' \tilde{\phi}_a^\dagger \phi_s) Tr(\Delta_R^\dagger \Delta_R - \Delta_L^\dagger \Delta_L) + h.c. \quad (1)$$

with λ' and λ'' real and where $\tilde{\phi}_{a,s} \equiv \tau_2 \phi_{a,s}^* \tau_2$. We thus need to introduce an additional symmetry under which ϕ_a and ϕ_s transform differently such as Z_2 with ϕ_a odd (or anti-symmetric) and ϕ_s even (or symmetric) under Z_2 to prevent this term.

Under Z_2 we have $\phi_a \rightarrow -\phi_a$, $Q_{1L} \rightarrow -Q_{1L}$ and $Q_{1R} \rightarrow -Q_{1R}$ while all other quark and Higgs fields including ϕ_s are invariant under Z_2 . Note that one generation of quarks is chosen to be odd under Z_2 to permit Yukawa couplings with ϕ_a and the remaining generations. Z_2 ensures that all terms in the Higgs potential are real since non-real P and CP invariant terms such as (1) vanish due to it.

Note that any symmetry that sets the purely imaginary quartic couplings to zero will work as well and we

designate symmetries such as Z_2 as Strong CP solution helping symmetries or SCP symmetries for short.

We will now discuss the Higgs potential in further detail and show how the symmetry breaking can happen to give rise to real Higgs VEVs and a Higgs mass spectrum where all the additional Higgses beyond the standard model Higgs are naturally very heavy.

The most general Higgs potential invariant under $SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times P \times CP \times Z_2$ is of the form

$$V_{Inv.} = V_{tri}(\Delta_L, \Delta_R) + V_{bi}(\phi_s, \phi_a) + \sum_{z=a,s} V(\phi_z, \Delta_L, \Delta_R) \quad (2)$$

where due to Z_2 and gauge invariance there are no terms such as in (1) that involve both ϕ_s and ϕ_a as well as $\Delta_{L,R}$, and V_{bi} contains only quartic terms that involve an even number of both ϕ_s and ϕ_a . In the above $V(\phi_z, \Delta_L, \Delta_R) + V_{tri}(\Delta_L, \Delta_R)$, with $z = a$ or s , is the most general Higgs potential of the minimal left-right symmetric model involving only one bi-doublet ϕ_z which is given in references [14, 15]. V , V_{tri} and V_{bi} are also functions of the mass parameters and other coupling constants of the model and these indices have been suppressed. All these parameters are real due to CP and Z_2 that have been imposed.

We allow for the Higgs potential to have dimension 2 terms that break CP and Z_2 softly such as $-\mu_{1as}^2 Tr(\phi_a^\dagger \phi_s) + h.c.$. All the parameters of soft CP and Z_2 breaking terms such as μ_{1as}^2 are real due to P . We thus have for the Higgs potential

$$V_{Higgs} = V_{Inv.} + V_{Soft} \quad (3)$$

where the detailed form of V_{Inv} and V_{Soft} is given in Appendix A and has all real parameters.

The neutral component of the Higgs fields can pick up VEVs and these are indicated by

$$\langle \Delta_{L,R} \rangle = \begin{pmatrix} 0 & 0 \\ \delta_{L,R} & 0 \end{pmatrix}, \langle \phi_{a,s} \rangle = \begin{pmatrix} \kappa_{a,s} & 0 \\ 0 & \kappa'_{a,s} \end{pmatrix} \quad (4)$$

We break $SU(2)_R \times U(1)_{B-L}$ to $U(1)_Y$ in the usual way with the neutral component of Δ_R picking a large VEV $v_R \equiv \delta_R$ that sets the scale of parity breaking M_R . Due to $SU(2)_R$ invariance of the potential v_R can be chosen to be real and positive without any loss of generality. Note in equation (A1) that the coupling term $\rho_3 Tr(\Delta_R^\dagger \Delta_R) Tr(\Delta_L^\dagger \Delta_L)$ with ρ_3 positive has its lowest value if $\delta_L = 0$ and therefore this term can ensure that $v_R \gg |\delta_L|$. This is the usual way parity breaks in the left-right model.

We now look at the bi-doublet VEVs. Without loss of generality we can use $SU(2)_L$ invariance to choose κ_s to be real and positive. The remaining VEVs $\kappa'_s, \kappa_a, \kappa'_a$ and δ_L can in general be complex and their phases are determined by the minimization condition $\partial V_{Higgs} / \partial \phi = 0$ for all the scalar field components ϕ in the theory.

We now show that for a scalar potential with all real parameters the minimization condition is satisfied when the VEVs of the fields are all real.

We denote with subscripts *Re* and *I* the real and imaginary components of every field – for example $\kappa_a = \kappa_{aRe} + i\kappa_{aI}$ etc. V_{Higgs} can now be written as a sum of products of real and imaginary parts of the fields. However V_{Higgs} itself is a real-valued function as the Hermitian conjugate of every term is also present in it. Consider a term such as $i\mu_{1as}^2\kappa_{aI}\kappa_{sRe}$ with just one imaginary component of the field that can potentially be obtained from the term $-\mu_{1as}^2\text{Tr}(\phi_a^\dagger\phi_s)$. Since μ_{1as}^2 is real, the Hermitian conjugate $-i\mu_{1as}^2\kappa_{aI}\kappa_{sRe}$ obtained from $-\mu_{1as}^2\text{Tr}(\phi_s^\dagger\phi_a)$ provides a canceling contribution. Hence there can be no terms with an odd number of the imaginary components of the fields in the real-valued scalar potential that has all real parameters.

This implies that

$$\left. \frac{\partial V_{Higgs}}{\partial \phi_{iI}} \right|_{\phi_{1I}=\dots=\phi_{nI}=0} = 0 \quad (5)$$

This is because every term that depends on the imaginary components must have at least two of them and when the partial derivative is taken and evaluated at the point where all the imaginary components are set to zero it vanishes.

Thus we have shown that real VEVs satisfy the minimization conditions. We now show that there exists a region of parameter space where the scalar potential is at the minimum (and not only at an extremum) for real VEVs.

There are several terms in the Higgs potential that depend on the phases of the fields. We choose the sign of the parameter of every such term so that each of these terms makes a negative contribution to the Higgs potential when all the field values are real and positive. For example we choose mass parameters $\mu_{2s}^2, \mu_{2a}^2, \mu_{1as}^2, \mu_{2as}^2, \mu_{2sa}^2 \geq 0$ in (A2) and (A4), and coupling parameters such as $\lambda_{2s}, \lambda_{2a}, \lambda_{4s}, \lambda_{4a}, \lambda_{3as}, \alpha_{2a}, \alpha_{2s}, \rho_4 \leq 0$ in (A2) and (A3), as well as the β 's ≤ 0 in (A2).

With this choice of parameter space it is easy to see that the minimum of the Higgs potential will be attained when all the fields $\kappa_{a,s}, \kappa'_{a,s}$ and $\delta_{L,R}$ pick up real positive VEVs (and not any other phases) since each and every term of the potential is independently minimized with this choice.

Thus we have proved without any fine-tuning that there exists a region of parameter space where the VEVs are real. In reality the parameter space of real VEVs is much bigger – we need to evaluate the second derivatives of the scalar potential at the extremum point with real VEVs and diagonalize it. The condition that every mass eigenvalue so obtained is positive (unless it is zero because it is a would-be Goldstone mode) is the only condition that the parameters would need to satisfy for the extremum to be a minimum. In fact with only one bi-doublet it is known that a scalar potential with real

parameters will only have real VEVs (unless there is a lot of fine-tuning) [14]. With two bi-doublets as we now have, there maybe some space for non-real VEVs, but a significant region of parameter space will give rise to real VEVs. The regions with non-real VEVs are phenomenologically ruled out as they would give rise to a tree-level $\bar{\theta}$ and we are left with regions of parameter space with real VEVs.

The remaining minimization conditions $\partial V_{Higgs}/\partial \phi_{iRe} = 0$ have to be solved simultaneously to obtain the real VEVs in terms of the parameters of the potential.

Note that the natural value for mass parameters (μ) of all the dimension 2 terms of the Higgs potential in Appendix A is the parity breaking scale M_R , which is heavier than the electro-weak scale and can even be as large as the Planck scale. Thus the natural values for all the VEVs will be either zero or the scale M_R . To get a VEV at the electro-weak scale we need to fine-tune parameters in one combination of the minimization equations (say in $\partial V_{Higgs}/\partial \kappa_s = 0$) so that order M_R terms cancel giving rise to the weak scale. This is the usual fine-tuning that is needed in the minimal left-right symmetric model or in any non-supersymmetric theory owing to the problem of quadratic divergence. It ensures an electro-weak scale VEV for κ_s and one light Higgs boson which is identified with the standard model Higgs boson.

If we do not allow for any other fine-tuning the other Higgs bi-doublet ϕ_a cannot pick up a VEV in the absence of soft *CP* and Z_2 breaking terms in V_{soft} .

Therefore we have introduced V_{soft} as a perturbation so that terms such as $\mu_{1as}^2\text{Tr}(\phi_s^\dagger\phi_a) + hc$ will induce a VEV to ϕ_a from the VEVs of ϕ_s . Once we substitute for δ_R in the Higgs potential of Appendix A then the terms involving κ_a for example can be written in the form

$$V = -\mu_{1as}^2\kappa_a\kappa_s + m_{eff}^2\kappa_a^2 \quad (6)$$

where in order to illustrate the process we have set μ_{2as}^2, μ_{2sa}^2 in V_{soft} to zero. V_{bi} can be neglected, since for electroweak scale VEVs, V_{bi} is much weaker than V_{soft} . We have also neglected terms involving κ'_a such as $\kappa_a\kappa'_a$ for purposes of illustration. In the above m_{eff}^2 depends on terms such as μ_{1a}^2 in the Higgs potential of Appendix A and the VEV δ_R . Further, $\mu_{1as}^2, m_{eff}^2, \kappa_a$ and κ_s are all real.

Minimizing the above potential for κ_a gives $\kappa_a = (\mu_{1as}^2/2m_{eff}^2)\kappa_s$, where the magnitude of term in the brackets is < 1 as it was treated as a perturbation and can be as low as $\sim 10^{-4}$ (since κ_a is responsible for Yukawa couplings between first generation of quarks and the rest of the generations). We can see that even if $\mu_{1as}, m_{eff} \sim M_R$, κ_a is naturally small at the electroweak scale set by κ_s .

In general both κ_a and κ'_a as well as the rest of the terms in V_{soft} will be present. In the potential terms that are quadratic in the fields κ_a and κ'_a as in equation (6) will be the only relevant terms in the leading

order, and will include cross-terms such as $\kappa_a \kappa'_a$. Solving minimization equations simultaneously these VEVs can be obtained in terms of liner combinations of the first bi-doublet VEVs κ_s and κ'_s without any fine-tuning.

The case with one bi-doublet is considered in detail in reference [14]. We have shown how the second bi-doublet can naturally pick up VEVs from the first bi-doublet VEVs.

Since there is no fine-tuning other than what is usual for the standard model Higgs mass, the natural scale of all the other Higgs boson masses is $\sim M_R$.

δ_L picks up a VEV of the order v^2/v_R where v is the weak scale, owing to the β terms in the potential in equation (A2) in the Appendix A as is usual in the left-right symmetric model. The VEV induced for δ_L is also naturally real as the parameters of the potential are real.

In the next section we look at the fermion mass matrices which are obtained from the Yukawa matrices and real VEVs.

In place of V_{soft} we can also use a complex singlet to spontaneously break CP without need for any dimension 2 soft CP breaking terms. This is discussed in section III F. In the next section we show how real VEVs lead to the solution of the strong CP problem as well as to a maximal CP Phase.

C. No Strong CP Phase, Maximal CP Phase!

Since Yukawa matrices are Hermitian due to parity, real Higgs VEVs imply that quark mass matrices M_u and M_d are also Hermitian and $\bar{\theta} = \text{ArgDet} M_u M_d$ vanishes and the strong CP problem is solved at the tree level. This can be explicitly seen below from the Hermitian form of the mass matrices in equation (9) obtained using (7) and (8)

To note how the maximal CP phase arises recall that under CP $\phi_a \rightarrow -\phi_a^*$, while the rest of the fields transform in the usual way as given in section III A. CP conservation implies that the Yukawa couplings involving ϕ_a have to be purely imaginary while the rest of the Yukawa couplings are real. Since the VEVs are real this implies that a purely imaginary contribution is made to the quark mass matrix by Yukawas involving ϕ_a .

Moreover, the same quark pairs coupling to ϕ_a cannot also couple to ϕ_s since ϕ_a and ϕ_s transform differently under Z_2 (or in general under any SCP). That is if the purely imaginary Yukawa coupling $i\bar{Q}_{iL}\phi_a Q_{jR}$ is permitted by Z_2 , for the same i, j we cannot also have the real Yukawa term $\bar{Q}_{iL}\phi_s Q_{jR}$. This implies that after symmetry breaking through real VEVs for ϕ_a and ϕ_s , the quark mass matrix will not only be Hermitian but will also have entries that are either purely real or purely imaginary. Thus in the symmetry basis we see that CP violation is maximal – ie the phase is $\pi/2$. It is interesting that the vanishing of tree-level $\bar{\theta}$ (which motivated the Z_2 symmetry) implies a maximal phase for CP violation in the quark mass matrix in the symmetry basis.

Concretely if the first generation quarks are odd under Z_2 and the rest are even, the most general Yukawa terms invariant under $P \times CP \times Z_2$ take the form:

$$\begin{aligned} & \sum_{j=2-4} i[\bar{Q}_{1L}(\underline{h}_{1j}\phi_a + \tilde{h}_{1j}\tilde{\phi}_a)Q_{jR} - \bar{Q}_{jL}(\underline{h}_{1j}\phi_a + \tilde{h}_{1j}\tilde{\phi}_a)Q_{1R}] \\ & + \bar{Q}_{1L}(\underline{h}_{11}\phi_s + \tilde{h}_{11}\tilde{\phi}_s)Q_{1R} + \bar{Q}'_L(\underline{h}'\phi_s^\dagger + \tilde{h}'\tilde{\phi}_s^\dagger)Q'_R \\ & + \sum_{i,j=2-4} \bar{Q}_{iL}(\underline{h}_{ij}\phi_s + \tilde{h}_{ij}\tilde{\phi}_s)Q_{jR} + h.c. \end{aligned} \quad (7)$$

where in the last term $\underline{h}_{ij} = \underline{h}_{ji}^*$, $\tilde{h}_{ij} = \tilde{h}_{ji}^*$ due to P , and $\underline{h}_{ij}, \tilde{h}_{ij}$ are real due to CP for any i, j in all terms. The underline on the Yukawa terms like \underline{h}_{ij} has been used to signify that these Yukawas are in the symmetry basis.

Due to vectorlike quarks there are also the direct quark mass terms

$$M_i \bar{Q}_{iL} Q'_R + M_i^* \bar{Q}'_L Q_{iR} + h.c \quad (8)$$

where the form is due to parity and $M_i = M_i^*$ are real due to CP invariance. As we can see in (9) below $M_1 \neq 0$ is needed to ensure that the CP phase generated is not trivially rotated away. M_1 breaks Z_2 softly.

Using (7) and (8) after electro weak symmetry breaking the up sector quark mass terms are given by $(\bar{u}_L, \bar{u}'_L) M_u \begin{pmatrix} u_R \\ u'_R \end{pmatrix}$ with

$$M_u = \begin{pmatrix} \underline{h}_{11}^u v & i\underline{h}_{12}^u v & i\underline{h}_{13}^u v & i\underline{h}_{14}^u v & M_1 \\ -i\underline{h}_{12}^u v & \underline{h}_{22}^u v & \underline{h}_{23}^u v & \underline{h}_{24}^u v & M_2 \\ -i\underline{h}_{13}^u v & \underline{h}_{23}^u v & \underline{h}_{33}^u v & \underline{h}_{34}^u v & M_3 \\ -i\underline{h}_{14}^u v & \underline{h}_{24}^u v & \underline{h}_{34}^u v & \underline{h}_{44}^u v & M_4 \\ M_1 & M_2 & M_3 & M_4 & \underline{h}'^u v \end{pmatrix} \quad (9)$$

where \bar{u}_L is the 1×4 row vector $(\bar{u}_{1L}, \dots, \bar{u}_{4L})$ and from (7), $\underline{h}_{1j}^u v = \underline{h}_{1j}^u \kappa_a + \tilde{h}_{1j}^u \kappa'_a$ for $j \neq 1$, $\underline{h}_{ij}^u v = \underline{h}_{ij}^u \kappa_s + \tilde{h}_{ij}^u \kappa'_s$ for i and $j \neq 1$ as well as for $i = j = 1$, and $\underline{h}'^u v = \underline{h}'^u \kappa_s + \tilde{h}'^u \kappa'_s$. Note that $\{\kappa_{a,s}, \kappa'_{a,s}\}$ are the diagonal elements of $\langle \phi_{a,s} \rangle$ and are real, and $v^2 = \kappa_a^2 + \kappa'_a{}^2 + \kappa_s^2 + \kappa'_s{}^2$. Note that we are using a convention where $v \sim 173 \text{ GeV}$ so that a $\sqrt{2}$ has been absorbed in its definition.

The down quark mass matrix M_d is similar to the up quark mass matrix with $u \rightarrow d$ and $\kappa_{a,s} \leftrightarrow \kappa'_{a,s}$ in equations determining them. We use the notation \underline{H}_u for the Hermitian upper 4×4 sub-matrix of M_u in (9) divided by the VEV v . Likewise \underline{H}_d is the upper 4×4 submatrix of M_d divided by v . The underline once again indicates that these matrices are in the symmetry basis.

Note that the above matrix has three light quark mass eigenvalues at the weak scale corresponding to the usual 3 light chiral families and 2 heavy quark mass eigenvalues $\sim \pm \sqrt{\sum |M_i|^2}$ (up to electro-weak corrections) – thus there are two heavy down quarks one each from the vectorlike doublet and the vector like singlet family. Similar is the case with the up sector.

D. Role Played by P, CP , and Z_2

To summarize,

- P implies that the Yukawa matrices are Hermitian.
- CP implies that the Yukawa parameters are either purely imaginary (if they couple to ϕ_a) or purely real (if they couple to ϕ_s).
- In addition to CP , Z_2 is needed to ensure that there are no non-real parameters in the scalar potential. Parameters of dimension 2 soft CP and Z_2 breaking terms are real due to P . Since all parameters of the Higgs potential are real all VEVs can be naturally real.
- The quark mass matrices that depend on the product of Hermitian Yukawas with real VEVs are also Hermitian thus ensuring that $\arg \det M_u M_d = 0$ and solving the strong CP problem.
- Real VEVs multiplying the purely imaginary or real Yukawa terms give rise to purely imaginary or real quark mass matrix terms, simultaneously ensuring a maximal CP phase in the quark mass matrices in the symmetry basis. Here Z_2 ensures that the same quark pairs do not couple to both ϕ_a and ϕ_s , keeping the purely imaginary and real terms separate.
- The vectorlike quarks ensure that the CP phase generated cannot be trivially rotated away.
- Real VEVs conserve either P or CP or both. For example real VEVs of $\Delta_{L,R}$ conserve CP while breaking P , real VEVs of ϕ_a conserve P while breaking CP , and real VEVs of ϕ_s conserve both P and CP .

E. Spontaneous CP Phase in Fermion Sector

An interesting feature of equation (7) is that we can redefine the first generation quark fields $Q_{1L,R} \rightarrow -iQ_{1L,R}$ so that the phase i gets absorbed so that it disappears from the Yukawa terms in (7) and appears in terms of the redefined fields in the dimension 3 mass term as $iM_1(\bar{Q}_{1L}Q'_{1R} - \bar{Q}'_{1L}Q_{1R})$.

Also since all terms of the Higgs potential are real and the Higgs VEVs are also real, the model has a complex phase only in the dimension 3 fermion mass term and at this stage is almost exactly like the model of [6] where CP is broken softly by fermion mass terms. The only difference is that it is more predictive – CP phase has been determined to be $\pi/2$ and CP violation is actually spontaneous since there is a way of defining CP so that the Lagrangian is CP symmetric, as that's how we constructed the model. However at the tree-level the spontaneous phase is generated in the quark mass matrix and not in the Higgs terms.

That spontaneous CP violation in multi-Higgs doublet models can appear at the tree level in the fermion sector instead of the Higgs sector was noticed in reference [16] in a more academic context and its relevance to physical things like the strong CP problem and a maximal CP phase was not considered.

F. Domain Wall Problem

Spontaneous CP violation at the weak scale (due to VEV of ϕ_a) can lead to the domain wall problem [17]. We have addressed this automatically since we have dimension 2 soft CP breaking terms V_{soft} . Moreover, the scale of the mass parameters in V_{soft} is naturally of the order M_R .

However if we do not want any soft CP breaking, we can add complex singlet fields σ_a that can pick up large CP violating real VEVs that conserve P , such that under CP $\sigma_a \rightarrow -\sigma_a^*$, under P $\sigma_a \rightarrow \sigma_a^*$ and under Z_2 $\sigma_a \rightarrow -\sigma_a$. As before Z_2 ensures that dangerous P and CP symmetric terms of the type $i\sigma_a(\Delta_R^\dagger \Delta_R - \Delta_L^\dagger \Delta_L) + h.c.$ are absent. Further if we change the CP transformation of ϕ_a to be $\phi_a \rightarrow \phi_a^*$ then the maximal CP phase is generated due to the real VEV of σ_a from the Yukawa term $i h_a \sigma_a (\bar{Q}_{1L} Q'_{1R} - \bar{Q}'_{1L} Q_{1R}) + h.c.$. The terms such as $\mu_{1as}^2 \text{Tr} \phi_a^\dagger \phi_s + h.c.$ with μ_{1as}^2 real, now conserve CP while breaking Z_2 softly. Thus with such a singlet, CP can be violated only spontaneously while Z_2 can be broken by dimension 2 terms and conserved by dimension 3 and 4 terms. As before Z_2 ensures that the real and purely imaginary contributions do not mix in the symmetry basis in the quarks mass matrix and the CP violation is maximal. If σ_a picks a large VEV higher than the scale of inflation then the domain walls will be washed away resolving the problem.

The case of a real singlet has already been considered in [6] and it does not require a Z_2 symmetry to be imposed as the above dangerous term is automatically absent if σ_a is real and the strong CP problem is solved just by P and CP symmetries without needing additional help.

In what follows we allow real $\mu_{1as}^2 \neq 0$ and CP is either broken softly without a singlet, or equivalently there can be the singlet and CP is broken spontaneously without any soft CP violation. The discussion in the rest of the paper does not depend on the details of this choice.

IV. RADIATIVE CORRECTIONS AND NEUTRON EDM

We follow the procedure in reference [6] where radiative corrections to the quark mass matrix and consequently to $\bar{\theta} = \text{ArgDet} M_u M_d$ have been evaluated at the one loop level.

Since P and CP violating terms will interact with one another in loops we expect $\bar{\theta}$ to be generated radiatively.

Without loss of generality we can choose a flavour basis via a common 4×4 orthogonal transformation O on the up and down, such that $O_{ij}^T M_j = \delta_{i4} M$ where $\delta_{i4} = 0$ for $i < 4$ and $\delta_{44} = 1$. In the transformed basis H_u and H_d are hermitian matrices with complex phases as they are obtained from Hermitian matrices in symmetry basis by $H_{u,d} = O^T \underline{H}_{u,d} O$. Correspondingly the mass matrices

in (9) transform as $M_{u,d} \rightarrow \begin{pmatrix} O^T & 0 \\ 0 & 1 \end{pmatrix} M_{u,d} \begin{pmatrix} O & 0 \\ 0 & 1 \end{pmatrix}$.

We can now apply a common unitary rotation in the light 3×3 sector to diagonalize the 3×3 subspace of H_u and rotate away all phases in the 4th row (and column) so that it is real symmetric. In this basis H_d is a 4×4 hermitian matrix with complex coefficients. Writing explicitly the matrix elements $h_{ij}^{u,d}$ of $H_{u,d}$, the up and down mass matrices look as follows in this physical basis:

$$M_u = \left(\begin{array}{cccc|c} h_{11}^u v & 0 & 0 & h_{14}^u v & 0 \\ 0 & h_{22}^u v & 0 & h_{24}^u v & 0 \\ 0 & 0 & h_{33}^u v & h_{34}^u v & 0 \\ h_{14}^u v & h_{24}^u v & h_{34}^u v & h_{44}^u v & M \\ 0 & 0 & 0 & M & h_{4u}^u v \end{array} \right) \quad (10)$$

$$M_d = \left(\begin{array}{cccc|c} h_{11}^d v & h_{12}^d v & h_{13}^d v & h_{14}^d v & 0 \\ h_{12}^{d*} v & h_{22}^d v & h_{23}^d v & h_{24}^d v & 0 \\ h_{13}^{d*} v & h_{23}^{d*} v & h_{33}^d v & h_{34}^d v & 0 \\ h_{14}^{d*} v & h_{24}^{d*} v & h_{34}^{d*} v & h_{44}^d v & M \\ 0 & 0 & 0 & M & h_{4d}^d v \end{array} \right) \quad (11)$$

where in this basis M_u is real symmetric and M_d is Hermitian with complex coefficients $h_{ji}^d = h_{ij}^{d*}$.

We denote by $\delta M_{u_{ij}}$ the radiative corrections to the i^{th} row and j^{th} column of M_u and evaluate the determinant to the lowest order in δM_u and find as in reference [6] the most significant contribution to

$$\arg \det(M_u + \delta M_u) = Im \sum_{i=u,c,t} \frac{\delta M_{u_{ii}}}{h_{ii}^u v} \quad (12)$$

$\delta M_{u_{ii}}$ to one loop was evaluated in reference [6] and we outline the steps here leading to the same results. We keep the discussion more general so that the same results also apply to the case of more than one bi-doublet. Note that in reference [6] we were working in a basis where the 3×3 sector of M_d was diagonal and so we actually evaluated the contributions to $\delta M_{d_{ii}}$ (instead of $\delta M_{u_{ii}}$) in that work. However the procedure is the same when we work in the basis where 3×3 sector of M_u is diagonal.

To generate an imaginary part to $\delta M_{u_{ii}}$ we need to look for Feynman diagrams where there are both complex phases and parity violation.

In the Feynman gauge the charged would be goldstone boson G_L^+ corresponding to $SU(2)_L$ breaking makes a logarithmically divergent contribution to θ owing to the

process in Figure 1. Since it is the goldstone mode of $SU(2)_L$ transformation its Yukawa couplings are [15]

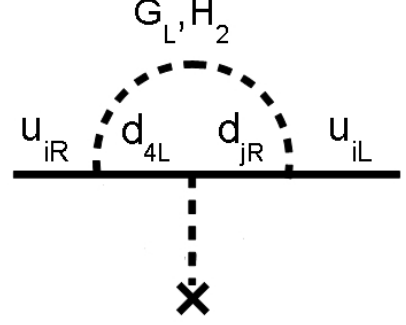


FIG. 1. Divergent part of radiative contributions from charged would be Goldstone mode and second Higgs doublet that are related by parity cancels

$$\mathcal{L}_Y = \bar{u}_{iR} h_{ij}^u d_{jL} G_L^+ - \bar{u}_{iL} h_{ij}^d d_{jR} G_L^+ + h.c. \quad (13)$$

However, since among the Higgses only $\phi_{a,s}$ couple to the quarks, and they pick up P symmetric real VEVs, and the Yukawa matrices are Hermitian, there cannot be a net one-loop contribution to $\bar{\theta}$ from the Higgses in the limit that the bidoublets do not couple to Δ_R – whose VEV is the only source of parity violation.

Therefore the contribution of G_L^+ to $\bar{\theta}$ will be exactly canceled by another massless mode in the Higgs bidoublets which we can call H_2^+ , and it corresponds to a global $SU(2)_R^{bi}$ rotation U_R^{bi} of the bi-doublets and fermions alone. That is, under this unitary transformation, $\phi_{a,s} \rightarrow \phi_{a,s} U_R^{bi\dagger}$, $Q_{iR} \rightarrow U_R^{bi} Q_{iR}$, and $\Delta_R \rightarrow \Delta_R$. In the symmetry limit Δ_R Higgses do not couple to the bidoublets other than through $Tr(\phi_s^\dagger \phi_s) Tr \Delta_R^\dagger \Delta_R$, which due to the trace does not convey $SU(2)_R$ breaking to the bi-doublets.

Now if we turn on the P, CP and Z_2 invariant coupling terms between the bidoublet and $\Delta_{L,R}$ such as $\alpha_{3s} Tr(\phi_s^\dagger \phi_s \Delta_R^\dagger \Delta_R + \phi_s \phi_s^\dagger \Delta_L^\dagger \Delta_L) + h.c.$ of equation (A2) so that U_R^{bi} is explicitly broken, then when Δ_R picks up a large VEV, H_2^+ will acquire a large mass $M_{H_2^+} \sim \sqrt{\alpha_{3s}} M_R$, where M_R is the parity breaking right-handed scale. The cancellation of the divergent part of the contribution from G_L^+ by H_2^+ is unaffected by the mass picked up by H_2^+ and a finite contribution to $\delta M_{u_{ii}}$ is generated due to figure 1 which depends on the product $(H_d H_d H_u)_{ii}$ so that $\bar{\theta}$ due to (12) becomes

$$\bar{\theta}|_{\delta M_u} \sim \frac{1}{16\pi^2} \sum_{\substack{i=1 \text{ to } 3 \\ j=1 \text{ to } 4}} Im \left(\frac{h_{ij}^d h_{j4}^d h_{4i}^u}{h_{ii}^u} \right) \ln(M_{H_2^+}/M) \quad (14)$$

where $M < M_{H_2^+}$ and the logarithmic dependence is because the divergence of figure 1 beyond M has been canceled beyond scale $M_{H_2^+}$.

As in reference [6] we also note that if $M > M_{H_2^+}$, then since below the scale M we have the left-right symmetric model, parity protects $\bar{\theta}$ from being generated so that the logarithm above is replaced by the factor $(M_{H_2^+}/M)^2$ which vanishes in the limit the parity breaking scale $M_{H_2^+} \rightarrow 0$.

Note that the bounds on $\bar{\theta}$ and d_n we are calculating are on their magnitude and so the sign of the quantities is not relevant for these purposes and we are not keeping track of it. Also the bounds are an order of magnitude estimate and not exact. For example there can be a quark mass dependent coefficient of order 1 that multiplies the terms within the summation sign of (14). That is, the coefficient for $j = 4$ in (14) when the heavy quark is in the inner loop of figure 1 can be different from those for $j = 1 - 3$ for light quarks whose masses can be taken to be zero. However since these coefficients are both order 1 we have combined the two terms instead of writing them separately. This simplification does not affect our estimates of the lower bounds.

Keeping these in mind we write in a compact form

$$\bar{\theta}|_{\delta M_u} \sim \frac{1}{16\pi^2} \sum_{\substack{i=1 \text{ to } 3 \\ j=1 \text{ to } 4}} \text{Im} \left(\frac{h_{ij}^d h_{j4}^d h_{4i}^u}{h_{ii}^u} \right) \ell \quad (15)$$

where the factor

$$\ell = \begin{cases} \ln(M_{H_2^+}/M) \geq 1 & \text{for } M < M_{H_2^+} \\ (M_{H_2^+}/M)^2 & \text{for } M > M_{H_2^+} \end{cases} \quad (16)$$

The choice of the symbol ℓ is to remind us that its a *logarithmic* factor for $M < M_{H_2^+}$ and for purposes of calculating lower bounds in this region of parameter space can be chosen to be $O(1)$. We also note that (15) is in the basis where 3×3 submatrix of M_u is diagonal. It can also be written in a basis independent manner as shown in reference [6] as

$$\bar{\theta}|_{\delta M_u} \sim \frac{1}{16\pi^2} \sum_{\substack{i,l=1 \text{ to } 3 \\ j,k=1 \text{ to } 4}} \text{Im} \left(h_{ij}^d h_{jk}^d h_{kl}^u (h_{3 \times 3}^{-1})_{li} \right) \ell \quad (17)$$

where $h_{3 \times 3}^{-1}$ is the inverse of the 3×3 submatrix of H_u . The contribution to $\bar{\theta}$ from radiative corrections to M_d can be got from (17) with $u \leftrightarrow d$.

Note that the Yukawa terms $L_R^T \tau_2 \Delta_R L_R$ that give Majorana mass to the right handed neutrinos break $SU(2)_R^{bi}$ and α_3 cannot naturally be kept small. Therefore we expect $M_{H_2^+} \sim M_R$.

A. Neutron EDM With Maximal CP Violation

We will now use the formulas in (15) and (17) to evaluate the lower bound for $\bar{\theta}$ in terms of the CKM parameters. As each element of M_u and M_d in the symmetry

basis of (9) can potentially contribute to $\bar{\theta}$ we can get the lower bound by setting maximum number of these matrix elements to zero while still having sufficient non-zero off-diagonal elements to obtain the correct CKM mixing angles.

Since we need non-trivial participation from vector-like quarks for CP violation we need to set $M_1, M_4 \neq 0$. In addition we need 3 off diagonal Yukawa couplings $\underline{h}_{ij}^{u,d}$ to obtain the 3 CKM angles if we set $M_2 = M_3 = 0$. At least one these Yukawa terms (but not all) must be from the fourth row or column, or else the CP phase can be rotated away.

At this stage once we rotate the heavy quarks away and go into the physical basis of equations (10) and (11), the light 3×3 quark sector will have 3 off diagonal elements (they could be anywhere in up and down sectors) each coming from a different non-zero Yukawa of the symmetry basis. So a maximal CP phase $\pi/2$ is transferred to the light 3×3 sector and the CKM angle generated is too high at $\gamma \sim \pi/2$. To get the CKM phase of 68.8° , we need one more non-zero Yukawa, so finally we need a texture with 4 non-zero off-diagonal Yukawa terms in the symmetry basis.

For all such textures we can evaluate $\bar{\theta}$ and we find the lower bound $\bar{\theta} \geq 10^{-11} \ell$. For example, let us take all off-diagonal Yukawas as well as \underline{h}_{44}^u in M_u of (9) to be zero. Let us also take $\underline{h}_{44}^d = \underline{h}_{14}^d = \underline{h}_{24}^d = 0$ so that the only non-zero off-diagonal Yukawas in M_d are $\underline{h}_{12}^d, \underline{h}_{13}^d, \underline{h}_{23}^d$, and \underline{h}_{34}^d . Also we set $M_1, M_4 \neq 0$ and $M_3 = M_4 = 0$.

We now go into the physical basis by an orthogonal transformation in the $1 - 4$ plane with $s_1 = \sin\theta_1 = M_1/M$, $c_1 = \cos\theta_1$ and $M = \sqrt{M_1^2 + M_4^2}$ we obtain

$$H_u = \begin{pmatrix} c_1^2 \underline{h}_{11}^u & 0 & 0 & -s_1 \underline{h}_{11}^u \\ 0 & \underline{h}_{22}^u & 0 & 0 \\ 0 & 0 & \underline{h}_{33}^u & 0 \\ -s_1 \underline{h}_{11}^u & 0 & 0 & s_1^2 \underline{h}_{11}^u \end{pmatrix} \quad (18)$$

$$H_d = \begin{pmatrix} 0 & ic \underline{h}_{12}^d & ic_1 \underline{h}_{13}^d + s_1 \underline{h}_{34}^d & 0 \\ \star & \underline{h}_{22}^d & \underline{h}_{23}^d & is_1 \underline{h}_{12}^d \\ \star & \star & \underline{h}_{33}^d & is_1 \underline{h}_{13}^d + c_1 \underline{h}_{34}^d \\ 0 & \star & \star & 0 \end{pmatrix} \quad (19)$$

where the \star in the lower triangle is a short hand notation to indicate that the term is obtained from the corresponding element in the upper triangle by complex conjugation to make the matrix Hermitian. By comparing (18) and (19) with equations (10) and (11) we can get the Yukawa's in the physical basis ($h_{ij}^{u,d}$) in terms of those in the symmetry basis ($\underline{h}_{ij}^{u,d}$). Now using (15) we find the contribution to $\bar{\theta}$ from radiative corrections of M_u is:

$$\bar{\theta} \sim \frac{1}{16\pi^2} \text{Im} \left(\frac{h_{14}^u h_{43}^d h_{31}^d}{h_{11}^u} \right) \ell$$

$$\sim \frac{1}{16\pi^2} \left(\frac{-s_1 \underline{h}_{11}^u \underline{h}_{34}^d \underline{h}_{13}^d}{\underline{h}_{11}^u} \right) \ell \quad (20)$$

where we have dropped terms of order s_1^2 and taken $c_1 \sim 1$. Since the light 3×3 up sector is diagonal in (18), we have $s_1 \underline{h}_{34}^d \sim V_{ub} \underline{h}_{33}^d$ and $\underline{h}_{13}^d / (s_1 \underline{h}_{34}^d) \sim \cos \gamma$ where from the Particle Data Group's global CKM fits [18] we have $V_{ub} \sim 3.47 \times 10^{-3}$ and $\gamma \sim 68.8^\circ$ is approximately the CKM CP phase. Substituting in (20) we get

$$\bar{\theta} \sim \frac{1}{16\pi^2} \frac{m_b^2}{m_t^2} V_{ub}^2 \cos \gamma \ell \sim 10^{-11} \ell \quad (21)$$

where $\underline{h}_{33}^d = m_b/m_t \sim 1/40$ is the bottom quark Yukawa.

We similarly find for all choices of the placement of the non-zero off diagonal elements that the strong CP phase generated has the lower bound $\bar{\theta} \geq 10^{-11}$.

While making the above statement we note that there is a special case with the choice $\underline{h}_{i4}^{u,d} = 0$ in equation (9) for all $i = 1$ to 4 as it can lead to a vanishing $\bar{\theta}$ at the one-loop level. However this texture does not generate the observed CKM phase. With $M_i \neq 0$ in (8) and (9) this can be easily seen by first rotating in the $1-4$ plane, then in the $2-4$ and finally in the $3-4$ plane and going to the physical basis of (10) and (11). Due to the hierarchy of the Yukawa couplings and Yukawas involving the first generation being the smallest, there is no significant mixing generated by the above rotations in the light 3×3 sector between Yukawa terms in the first row and those in the second and third rows and therefore the CKM phase generated is too small. This texture is thus not easily viable.

An interesting thing about equation (20) is that the Yukawas involving the fourth heavy quarks such as \underline{h}_{34}^d are multiplied by the angle s_1 so that the product leads to a Yukawa term of the light first generation. This means that \underline{h}_{34}^d itself does not have to be small, it is the product $s_1 \underline{h}_{34}^d$ that is small.

Radiative corrections to M_d can also contribute but they turn out to be much smaller for textures with maximal CP such as those in (18) and (19). However they are relevant if the CP phase is not maximal as we will see in the next section.

B. Neutron EDM Bound for General CP Phase

If CP is broken softly by dimension 3 quark mass terms (ie) by choosing M_i to be complex or spontaneously via a real singlet (both these cases are studied in reference [6]) then the CP phase in the quark mass matrix in the symmetry basis can be non-maximal and arbitrary. We can then choose it to be close to the CKM phase γ . This means that with just 3 non-zero off-diagonal Yukawa elements in the symmetry basis (and $M_2 = M_3 = 0$) we can obtain the needed CKM angles and the CKM CP phase. For such textures, since there is one less non-zero term than in IV A, we find that the $\bar{\theta}$ generated will be

smaller by about two orders of magnitude and accordingly the lower bound is reduced.

To see this note that if the CP phase i in the first row of equation (19) is replaced by $e^{i\gamma'}$ with γ' close to the CKM phase γ , then we can set \underline{h}_{13}^d to zero and still obtain the needed CKM angles and γ . However if $\underline{h}_{13}^d = 0$ then the contribution to $\bar{\theta}$ from equation (20) vanishes.

On the other hand we find that the contribution to $\bar{\theta}$ from radiative corrections to M_d remain non-vanishing. Since the light 3×3 sector of M_d is non-diagonal we need to use the basis independent equation 17 with $u \leftrightarrow d$ to estimate them. We find

$$\begin{aligned} \bar{\theta} &\sim \frac{1}{16\pi^2} \text{Im} \left(\frac{h_{12}^d h_{23}^d h_{34}^d h_{14}^u h_{11}^u}{h_d h_s h_b} \right) \ell \\ &\sim \frac{\sin \gamma}{16\pi^2} \left(\frac{h_{12}^d h_{23}^d h_{34}^d s h_{11}^u}{h_d h_s h_b} \right) \ell \end{aligned} \quad (22)$$

where the product of the Yukawas in the denominator corresponds to the determinant of the light 3×3 down-sector Yukawa matrix, that is $h_{d,s,b} = m_{d,s,b}/m_t$. Substituting $\underline{h}_{12}^d = V_{us} \underline{h}_{22}^d$, $\underline{h}_{23}^d = V_{cb} \underline{h}_{33}^d$ and as before $s \underline{h}_{34}^d = V_{us} \underline{h}_{33}^d$ we get

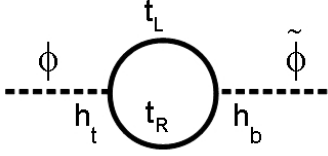
$$\bar{\theta} \sim \frac{1}{16\pi^2} \left(\frac{m_u^2 m_b}{m_d m_t^2} \right) V_{us} V_{cb} V_{ub} \sin \gamma \ell \sim 10^{-13.5} \ell \quad (23)$$

There is one more potential source of radiative corrections at the one-loop level that is relevant. In order to calculate lower bounds we have assumed textures with zeros in mass matrices which are not necessarily supported by any symmetries and therefore small non-zero values will be radiatively generated. To avoid fine-tuning, the natural tree-level values for these elements must be greater than the one-loop corrections.

Since the bi-doublets will acquire VEVs $\text{diag} \{ \kappa, \kappa' \}$ if a matrix element \underline{h}_{ij}^d is non-zero then the corresponding element \underline{h}_{ij}^u must be at least of the order $\underline{h}_{ij}^d \kappa' / \kappa$ (with $\kappa' \ll \kappa$) since both the up and down sector can receive contribution from a term such as $\tilde{h} \bar{Q}_{iL} \tilde{\phi} Q_{jR}$. The same is also true if the up sector has a non-zero element the corresponding down element will also get a small VEV by the same argument.

Even if κ' is set to zero it will acquire a one loop correction order $\kappa / (16\pi^2) (m_b/m_t) \sim \kappa / 6000$ due to the diagram in figure 2 where in the figure $h_t = \underline{h}_{33}$, $h_b = \underline{h}_{33}$ are the top and bottom quark Yukawas. Thus instead of zeros we should have a value for every element in the up (down) Yukawas that is at least $1/6000^{\text{th}}$ of the corresponding element in the down (up) Yukawas. Since this is very small it doesn't increase the lower bounds we have evaluated in (23).

However where this is relevant is for textures that may

FIG. 2. Radiative CP invariant contribution to κ'

have an additional zero in the diagonal element such as:

$$H_u = \begin{pmatrix} 0 & 0 & c_1 e^{i\gamma'} \underline{h}_{13}^u & 0 \\ 0 & \underline{h}_{22}^u & \underline{h}_{23}^u & 0 \\ \star & \star & \underline{h}_{33}^u & -s_1 e^{-i\gamma'} \underline{h}_{13}^u \\ 0 & 0 & \star & 0 \end{pmatrix} \quad (24)$$

$$H_d = \begin{pmatrix} 0 & s_1 \underline{h}_{24}^d & c_1 \epsilon & 0 \\ \star & \underline{h}_{22}^d & 0 & c_1 \underline{h}_{24}^d \\ \star & 0 & \underline{h}_{33}^d & -s_1 \epsilon^\star \\ 0 & \star & \star & 0 \end{pmatrix} \quad (25)$$

The light 3×3 sub-matrices of the above have a predictive texture [19] and for $\epsilon = 0$ it is known in the literature as the texture corresponding to solution 5 of Ramond, Roberts and Ross (RRR). This texture gives the approximate predictions $V_{us} = \sqrt{m_d/m_s}$ and $V_{ub} = \sqrt{m_u/m_t}$. If we now evaluate $\bar{\theta}$ using the above matrices we find that the contribution from radiative corrections to M_u as well as M_d vanishes if $\epsilon = 0$. Thus it maybe possible that the lower bound may be weaker than (23) for the case where CP violation is non-maximal, but we eliminate this possibility below.

As we have argued ϵ cannot be taken to be zero but must at least be $\epsilon \sim e^{i\gamma'} \underline{h}_{13}^u / 6000$ so as to prevent fine-tuning. Evaluating the contribution to $\bar{\theta}$ from radiative corrections to M_d by using equation (17) with $u \leftrightarrow d$ we get for the above ϵ

$$\begin{aligned} \bar{\theta} &\sim \frac{\sin(2\gamma')}{16\pi^2} \frac{s_1^2 \underline{h}_{24}^d \underline{h}_{13}^u \underline{h}_{33}^u}{6000 h_d h_s h_b} \ell \\ &\sim \frac{\sin(2\gamma')}{96000\pi^2} (V_{us}^2 V_{ub}^2) \frac{m_s m_t}{m_d m_b} \ell \\ &\sim 3 \times 10^{-10} \ell \end{aligned} \quad (26)$$

where we used $(s_1 \underline{h}_{24})/h_s = V_{us}$, $\underline{h}_{33}^u \sim h_t$, $\underline{h}_{13}^u/\underline{h}_{33}^u \sim V_{ub}$ and $h_{d,s,b,t} = m_{d,s,b,t}/m_t$ with the central values $V_{us} = 0.2253$, $V_{ub} = 0.00347$, $m_d = 5.1 \text{ MeV}$, $m_s = 100 \text{ MeV}$, $m_b = 4.19 \text{ GeV}$ and $m_t = 173 \text{ GeV}$.

Note that the above quantity came out higher than the previous cases because off-diagonal Yukawas in the up quark sector that give rise to any CKM mixing angles are 10 to 40 times higher than if the CKM angles resulted from a down quark Yukawa. And therefore θ is comparatively higher if up sector Yukawas are involved in generation of CKM mixing angles.

Thus we find that the lowest value we can get for the case of general (non-maximal) CP violation is given by (23) and this serves as the lower bound.

We have thus shown that for $M_{H_2^+} > M$ the lower bound for $\bar{\theta}$ is 10^{-13} for non-maximal CP violation and 10^{-11} for maximal CP violation. These bounds correspond to $d_n \geq 10^{-29} \text{ ecm}$ and $\geq 10^{-27} \text{ ecm}$ respectively. For $M_{H_2^+} < M$ there is a further suppression by factor $(M_{H_2^+}/M)^2$

We now see to what extent we can predict the CKM phase that can arise from the maximal phase.

V. CKM PHASE PREDICTION FROM TEXTURE AND FLAVOUR

Since a maximal CP phase is generated in the symmetry basis it is tempting to see if textures and flavour symmetries can help predict the CKM phase that is obtained from the maximal phase.

A. Using Texture

It is well-known that setting some of the diagonal elements of Yukawa matrix to zero will lead to a predictive mass-mixing angle relation, as the mass would then have to come from an off-diagonal element. We can use a similar trick to predict the CKM phase from texture alone.

As an example we take $\underline{h}_{11}^{u,d} = 0$ and take $M_2, M_3 = 0$ and take the four non-zero off diagonal Yukawas to be $\underline{h}_{12}^d, \underline{h}_{23}^d, \underline{h}_{34}^d$ and \underline{h}_{24}^u . We follow the same procedure as in section IV A.

Diagonalizing in the 1-4 plane so that the heavy quarks decouple (with $s = M_1/M$ as before), we get the 4×4 Yukawa matrices in the physical basis to be:

$$H_u = \begin{pmatrix} 0 & s \underline{h}_{24}^u & 0 & 0 \\ \star & \underline{h}_{22}^u & 0 & c \underline{h}_{24}^u \\ 0 & 0 & \underline{h}_{33}^u & 0 \\ 0 & \star & 0 & 0 \end{pmatrix} \quad (27)$$

$$H_d = \begin{pmatrix} 0 & i c \underline{h}_{12}^d & s \underline{h}_{34}^d & 0 \\ \star & \underline{h}_{22}^d & \underline{h}_{23}^d & i s \underline{h}_{12}^d \\ \star & \star & \underline{h}_{33}^d & c \underline{h}_{34}^d \\ 0 & \star & \star & 0 \end{pmatrix} \quad (28)$$

We can see from the top 2×2 sub-matrices in the above two matrices that we get the predictive relations $m_d \sim V_{us}^2 m_s$ (from (28) with $\underline{h}_{22}^d v = m_s$ and $V_{us} = c \underline{h}_{12}^d / \underline{h}_{22}^d$), and $\cos \gamma \sim (\sqrt{m_u/m_c})/V_{us}$ (from (27) with $\underline{h}_{22}^u v = m_c$ and $m_u = s^2 \underline{h}_{24}^u / \underline{h}_{22}^u$ and from (28)). Thus we get $\gamma \sim 79^\circ$ where we have used the central values $m_u = 2.5 \text{ MeV}$, $m_c = 1290 \text{ MeV}$, $V_{us} = 0.225$.

If we diagonalize the full 3×3 light quark sub-matrix and do the calculation more accurately γ can be a bit lowered to 76° . Comparing this with the experimental value of $\gamma = 68.8^\circ \pm 3^\circ$ we see that while the agreement is not too great, the texture does come pretty close and gives an idea of how the CKM matrix may get generated from the maximal phase. Of course once we turn on the terms we set to zero we can easily get corrections to the texture prediction and an agreement with experiments.

We evaluate the $\bar{\theta}$ for this texture and find it to be

$$\bar{\theta} = \frac{\sin \gamma}{16\pi^2} \frac{sh_{12}^d h_{24}^u m_s}{m_c} \ell \sim 10^{-11} \ell \quad (29)$$

We will now see how close we can get to predicting the CKM phase using flavour symmetries instead of just texture. In the next sub-section we motivate a Z_4 symmetry and evaluate how the maximal CP phase and Z_4 symmetry can help understand how the CKM phase may be generated.

B. CKM Phase Prediction Using Z_4 Flavour

If we take seriously the observed fact that the down quark mass m_d is related to the Cabibo angle via $m_d = V_{us}^2 m_s$, the two conditions that are needed to obtain this relation are:

- Quark Mass matrix is Hermitian.
- The down quark mass matrix has a near zero in its first diagonal entry.

Our strong CP solution ensures Hermiticity of the quark mass matrices and we impose an additional Z_4 symmetry to ensure the second condition.

Once Z_4 is imposed ϕ_s and $\bar{\phi}_s$ will transform differently under it and therefore both of them will not couple to the same quark pairs. This means that ignoring off-diagonal elements, the quark mass ratios will be the same for all generations since every generation would get its mass from ϕ_s and not from $\bar{\phi}_s$. Therefore if we fix $\kappa'_s/\kappa_s = m_b/m_t \sim 1/40$ then since $m_c/40$ and $m_u/40$ are much smaller than m_s and m_d , they would have to get their masses from off-diagonal terms. These constraints can lead to the predictions of CKM CP phase, and as expected the down quark mass.

Under Z_4 we have

$$\begin{aligned} Q_{1L,4L}, Q'_R &\rightarrow iQ_{1L,4L}, iQ'_R; & Q_{2L,3L} &\rightarrow -iQ_{2L,3L}; \\ Q_{2R,3R} &\rightarrow -Q_{2R,3R}; & \phi_s, \bar{\phi}_a &\rightarrow i\phi_s, i\bar{\phi}_a \end{aligned} \quad (30)$$

while the rest of the quark and Higgs fields are invariant. The leptons can also transform non-trivially under Z_4 but we do not write these out explicitly.

The most general Yukawa potential invariant under $P \times CP \times Z_2 \times Z_4$ is:

$$\sum_{i=1 \text{ to } 4} \underline{h}_{ii} \bar{Q}_{iL} \phi_s Q_{iR} + \underline{h}' \bar{Q}'_L \phi_s^\dagger Q'_R +$$

$$\begin{aligned} &\sum_{j=2 \text{ to } 3} [i\tilde{h}_{1j} (\bar{Q}_{1L} \tilde{\phi}_a Q_{jR} - \bar{Q}_{jL} \tilde{\phi}_a Q_{1R}) \\ &\quad + \tilde{h}_{j4} (\bar{Q}_{jL} \tilde{\phi}_s Q_{R4} + \bar{Q}_{L4} \tilde{\phi}_s Q_{jR})] \\ &\quad + i\tilde{h}_{14} (\bar{Q}_{1L} \phi_a Q_{R4} - \bar{Q}_{L4} \phi_a Q_{1R}) + h.c. \end{aligned} \quad (31)$$

where without loss of generality we have chosen a basis such that $\tilde{h}_{23} = 0$.

Once the diagonal elements of $\phi_{a,s}$ pick up real VEVs $\text{diag}\{\kappa_{a,s}, \kappa'_{a,s}\}$ the mass matrix M_d is given by

$$M_d = \begin{pmatrix} \underline{h}_{11}\kappa'_s & i\tilde{h}_{12}\kappa_a & i\tilde{h}_{13}\kappa_a & i\tilde{h}_{14}\kappa'_a & M_1 \\ -i\tilde{h}_{12}\kappa_a & \underline{h}_{22}\kappa'_s & 0 & \tilde{h}_{24}\kappa_s & M_2 \\ -i\tilde{h}_{13}\kappa_a & 0 & \underline{h}_{33}\kappa'_s & \tilde{h}_{34}\kappa_s & M_3 \\ -i\tilde{h}_{14}\kappa'_a & \tilde{h}_{24}\kappa_s & \tilde{h}_{34}\kappa_s & \underline{h}_{44}\kappa'_s & M_4 \\ M_1 & M_2 & M_3 & M_4 & \underline{h}\kappa'_s \end{pmatrix} \quad (32)$$

M_u is given by a similar matrix with $\kappa_{a,s} \leftrightarrow \kappa'_{a,s}$. Note that since we set $\kappa'_s/\kappa_s = m_b/m_t$, M_u is almost diagonal with negligible off-diagonal elements while the first two diagonal elements of M_d are small and need contribution from off-diagonal terms to provide the right masses for down and strange quarks.

When we make an orthogonal transformation in the $2-4$ plane by an angle $s_2 \sim M_2/\sqrt{M_2^2 + M_4^2}$ to go to the physical basis, the strange quark mass picks up an additional contribution so that it becomes

$$\begin{aligned} m_s^o &\sim \underline{h}_{22}\kappa'_s + 2s_2\tilde{h}_{24}\kappa_s \\ &\sim \frac{m_c m_b}{m_t} + 2s_2\tilde{h}_{24}\kappa_s \\ &\sim \pm \left| \frac{m_c m_b}{m_t} \right| + 2s_2\tilde{h}_{24}\kappa_s \end{aligned} \quad (33)$$

where in the last line we have a \pm since $m_c = \underline{h}_{22}\kappa$ and can be positive or negative depending on the sign of \underline{h}_{22} .

To keep matter simple we set $M_3 = 0$, $\underline{h}_{14} = 0$ and we rotate along the $1-4$ plane by an angle $s_1 = M_1/\sqrt{M_1^2 + M_2^2 + M_4^2}$. This takes us to the physical basis and the light 3×3 sector of M_d becomes:

$$M_{d3 \times 3} = \begin{pmatrix} 0 & i\tilde{h}_{12}\kappa_a + s_1\tilde{h}_{24}\kappa_s & i\tilde{h}_{13}\kappa_a + s_1\tilde{h}_{34}\kappa_s \\ \star & m_s^o & s_2\tilde{h}_{34}\kappa_s \\ \star & \star & m_b \end{pmatrix} \quad (34)$$

We will first do an approximate calculation. If $\underline{h}_{13} \ll s_1\tilde{h}_{34}$ then from (34) we get

$$s_1/s_2 \sim |V_{ub}/V_{cb}| \quad (35)$$

and using equation (33) with (35) we get

$$s_1\tilde{h}_{24}\kappa_s \sim \left| \frac{V_{ub}}{2V_{cb}} \right| \left(m_s^o \mp \left| \frac{m_c m_b}{m_t} \right| \right) \quad (36)$$

Substituting the above in (34) we get for the phase in the down quark mass matrix

$$\begin{aligned} \gamma^o &\sim \cos^{-1} \frac{s_1\tilde{h}_{24}\kappa_s}{V_{us}m_s^o} \\ &\sim \cos^{-1} \left[\left| \frac{V_{ub}}{2V_{cb}V_{us}} \right| \left(1 \mp \left| \frac{m_c m_b}{m_s m_t} \right| \right) \right] \sim 75.7^\circ \end{aligned} \quad (37)$$

where we have used from the particle data group's global fits to CKM matrix [18] $V_{us} = 0.2253$, $V_{cb} = 0.0410$, $V_{ub} = 0.00347$ and $m_s^0 \sim m_s = 100 \text{ MeV}$, $m_c = 1.290 \text{ GeV}$, $m_b = 4.19 \text{ GeV}$ and $m_t = 173 \text{ GeV}$. We have used the $+$ from \mp to get the lower value of γ° .

The phase γ° in the mass matrix M_d (in basis with M_u diagonal) is related to the CKM phase γ roughly by the relation $\gamma = \gamma^\circ - 3.4^\circ$ and so we find that due to Z_4 symmetry the maximal phase $\pi/2$ results in a CKM phase

$$\gamma = 72^\circ \pm \Delta(\gamma)_{\tilde{h}_{13}} \quad (38)$$

where

$$\Delta(\gamma)_{\tilde{h}_{13}} = \sin^{-1} \frac{\tilde{h}_{13} \kappa_a}{m_b V_{ub}} \quad (39)$$

is the contribution due to a small non-zero \tilde{h}_{13} .

There is a subtle point in the above equations. In principle \tilde{h}_{13} can also contribute to change the first term in (38) from 72° to a different number. This is because the exact value depends on equation (35) which will get modified once \tilde{h}_{13} is non-zero to

$$\frac{\sqrt{(\tilde{h}_{13} \kappa_a)^2 + (s_1 \tilde{h}_{34} \kappa_s)^2}}{s_2 \tilde{h}_{34} \kappa_s} \sim \left| \frac{V_{ub}}{V_{cb}} \right| \quad (40)$$

Expanding for small \tilde{h}_{13} we get

$$s_1/s_2 + O(\tilde{h}_{13}^2) = V_{ub}/V_{cb} \quad (41)$$

and thus there is no $O(\tilde{h}_{13})$ correction to the first term of equation (38) which remains 72° . Equation (38) with (39) is thus correct to $O(\tilde{h}_{13}^2)$.

The other Yukawa we set to zero \tilde{h}_{14} can be treated analogously and like \tilde{h}_{13} it does not change the 72° but just makes an additive contribution to the lowest order when its turned on.

Thus we see that the maximal phase generated in the quark mass matrix is actually changed to 72° on diagonalization to the physical mass basis due to known terms when Z_4 is imposed, while there can be a small correction from this value by unknown Yukawa terms that can be set to zero without affecting anything else. The value 72° is obtained by inputting central values from experiments for quark masses and CKM parameters (other than the CKM CP phase) and therefore it can have an experimental error of about 3° . This is thus consistent with the current experimentally determined value $\gamma = 68.8^\circ \pm 3^\circ$ even if Yukawas like \tilde{h}_{13} were too small to contribute any amount.

There are a couple of points we need to make before we proceed. Equations like (35) that relate CKM parameters to the symmetry basis Yukawas have been derived approximately so they can be presented simply but we can do a more exact derivation by using CKM matrix to $O(\lambda^4)$ in Wolfenstein parametrization. The resulting

equation (38) remains the same to within a degree when we do this. We also had set $M_3 \sim 0$ and when it is turned on it will also make a small contribution to equation (38) and so must be kept small. However including M_3 can also significantly contribute to $\bar{\theta}$ and M_3 can be bounded based on that.

We now evaluate $\bar{\theta}$ for this case using (15). We find the dominant contribution is due to radiative corrections to M_u and is

$$\begin{aligned} \bar{\theta} &\sim \left(\frac{1}{16\pi^2} \right) \left(\frac{\kappa_a}{\kappa_s} \right) s_1 \tilde{h}_{12} \tilde{h}_{24} \ell \\ &\sim \left(\frac{\cos \gamma}{16\pi^2} \right) \frac{V_{us}^2 m_s^2}{m_t^2} \ell \\ &\sim 3.5 \times 10^{-11} \ell \end{aligned} \quad (42)$$

We have used Z_4 symmetry as an example to show how the CKM phase and d_n may be calculated based on flavour symmetries. Other flavour symmetries can be similarly explored. Especially interesting would be those that are also motivated from the lepton sector considering the neutrino mixing.

VI. FEW COMMENTS

Before we conclude we make some comments:

1. In this work we have shown how P and CP symmetry breaking can lead to predictions for the neutrons electric dipole moment. For a large region of parameter space of the model these predictions turn out to be in the region that will be experimentally probed in the next few years. These experiments [3] are often considered to be complimentary to the LHC and are currently facing various hurdles [20]. However since the neutron EDM generated due to P and CP breaking does not get diminished even if there is no new physics up until the see-saw, GUT or Planck scales these experiments can in fact make a discovery due to physics at scales that are out of reach for LHC or future colliders. The last remaining parameter of the standard model $\bar{\theta}$ that gives rise to d_n , as well as neutrino masses and mixing, could both be a consequence of very high energy physics emerging from the left-right symmetric strong CP solving model.
2. Once CP is imposed, it will also apply on the lepton sector. If there are no vectorlike leptons (analogous to the vectorlike quarks) then CP phase is not generated at the tree level in the lepton sector, and there will be no CP phase in the neutrino mixing parameters. If there are vectorlike leptons then the same mechanism of CP generation can result in a maximal phase in the symmetry basis of the lepton sector (if CP is broken spontaneously by a complex singlet or softly by dimension 2 terms) or a general CP phase (if it is broken by real singlet

or softly by dimension 3 fermion mass terms). If there is Grand Unification, then we would expect vectorlike leptons to also be present.

3. Our model is generalizable to the supersymmetric left-right model (SUSYLR). In fact if we want to break CP spontaneously in SUSYLR so as to reduce the number of CP phases such as from the gaugino sector [21] that could generate too large a d_n , then we can introduce the vector-like quarks as we have done with similar consequences.
4. We have used a complete vectorlike quark family with $SU(2)_L$ doublet and parity related iso-singlet ($SU(2)_R$ doublet) quarks. In fact if only the Higgs triplets $\Delta_{L,R}$ that are the natural candidates for providing large Majorana masses to the right-handed neutrinos are used to break $SU(2)_R$, then this is the only choice of vectorlike quarks that works for solving the strong CP problem, since only they can have Yukawa couplings with bi-doublets and mix with the lighter quarks. So these quarks are uniquely picked in some sense.
5. The vectorlike quark masses are protected from quadratic divergences from the heavy sector since in the model without scalar singlets they only couple to the standard model Higgs VEVs (through Yukawa couplings with the bi-doublets) and through dimension 3 fermion mass terms. Thus these quarks can be naturally light including being at scales accessible by LHC and future colliders as pointed in our previous work [6]. Of course the dimension 3 mass terms can also be very heavy like being at the GUT scale. Their mass is an independent scale and can be light or heavy.

VII. CONCLUDING REMARKS

1950's and 60's presented the surprising discoveries that nature through weak interactions violates parity (P) and matter-antimatter (CP) symmetries. However while doing so nature has left us a puzzling situation of an extremely small CP violation by the strong interactions (as yet undetected) that seems to imply that there is an unknown hidden symmetry, along with a large CKM phase that seems to be put in by hand in the form of hard CP violation in the Yukawa and weak sector. If the unknown symmetries are in fact P and CP themselves which are spontaneously or softly broken and therefore hidden from our view, then we have shown in this work that the neutron must know about it since its electric dipole moment gets generated when these are broken.

Finding a neutron EDM consistent with the predictions in this work can provide evidence for P and CP symmetric laws of nature that would have been visible at high energies such as those at the time of the origin of the universe..

Moreover we find that the CP phases of Higgs VEVs that are generated spontaneously get determined by the P transformation properties since the strong CP solving vacuum is such that VEVs that violate CP must conserve P . This implies that for Higgses that couple to quarks and leptons the CP phases of the VEVs are maximal.

In addition if CP violation happens either softly by dimension 2 terms or spontaneously with a complex singlet we find that the CP phase generated in the quark (in general fermion) mass matrix is also maximal in the symmetry basis, thereby providing an understanding of the largeness of the CKM phase. The same symmetries such as Z_2 that are needed to ensure that there is no tree-level strong CP phase determine the maximality of the weak CP phases in the quark (fermion) mass matrix. While if CP is broken softly by dimension 3 fermion mass terms or spontaneously by a real singlet then the tree-level strong CP phase is automatically not present and no further symmetries like Z_2 are needed and as a result the CP violation in the quark (fermion) mass matrix need not also be maximal. In both cases vectorlike quarks (leptons) are needed to ensure the CP violation in the quark (lepton) matrices cannot trivially be rotated away.

The strong CP phase is radiatively generated at the one-loop level due to the $SU(2)_L$ breaking charged would be goldstone bosons and can be estimated from their Yukawa couplings with the quarks that include a full vectorlike quark family. P ensures that there is a mode in the Higgses of the right handed sector that cancels the divergences from the goldstone mode and a finite amount of $\bar{\theta}$ is generated that depends logarithmically on the mass ratio of the heavy higgs $M_{H_2^+}$ and the heavy quark mass M for the case where $M_{H_2^+} < M$.

The results for the $\bar{\theta}$ and d_n generated are:

- **For case of maximal CP violation:**

$$\bar{\theta} \geq 10^{-11} \times \begin{cases} \ln(M_{H_2^+}/M) \geq 1 & \text{for } M < M_{H_2^+} \\ (M_{H_2^+}/M)^2 & \text{for } M > M_{H_2^+} \end{cases} \quad (43)$$

- **For case of general CP violation:**

$$\bar{\theta} \geq 10^{-13} \times \begin{cases} \ln(M_{H_2^+}/M) \geq 1 & \text{for } M < M_{H_2^+} \\ (M_{H_2^+}/M)^2 & \text{for } M > M_{H_2^+} \end{cases} \quad (44)$$

and d_n can be obtained from the relation $d_n \sim 2 \times 10^{-16} \bar{\theta} \text{ ecm}$ [2].

If we are fortunate we will be able to observe the neutron EDM in the ongoing searches [3] that plan to probe $d_n \sim 10^{-26}$ to 10^{-27} ecm in the next few years and up to 10^{-29} ecm in the coming two decades. A lot depends on

whether $M < M_{H_2^+}$ or if its larger, then by how much? We know that the heavy doublet Higgs mass $M_{H_2^+}$ must be around the right-handed symmetry (parity) breaking scale. Based on the smallness of the neutrino mass this would most naturally be at or above the see-saw scale of $10^{14} GeV$ if we assume that the neutrino generations have similar Yukawa coupling with the standard model Higgs like their counterpart up sector quark generations. We also know that M being a fermion mass term is chirally protected and can be naturally small. Therefore there are reasonably good chances to observe the electric dipole moment of the neutron in the planned experiments that could provide evidence for the hidden P and CP symmetries in nature.

Equations (43) and (44) can also be used in the reverse so as to provide bounds for the mass ratio of the heavy higgs and heavy quarks based on non-observation of d_n . For example the experimental bound $\bar{\theta} < 10^{-10}$

implies from (43) for the case of maximal CP violation that the vectorlike quarks mass must be $M > e^{-10} M_{H_2^+} \sim M_{H_2^+}/20000$ so that the logarithm does not contribute more than a factor of 10. This together with $M_{H_2^+} \sim 10^{14} GeV$ would imply that such vectorlike quarks would be out of reach of the LHC if CP violation is maximal. However some recent work shows that the current experimental bound on $\bar{\theta}$ could be a factor of 5 weaker [22] based on the methods used to obtain the relationship between d_n and $\bar{\theta}$. If this is the case there is still a chance for the vectorlike quarks with maximal CP violation to be at the mass scales being probed by the LHC. Of course for general, non-maximal CP violation there is as yet no relevant constraint on the scale of the vectorlike quarks masses from neutron EDM experiments and they could be within the LHC reach or at very high scales.

Appendix A: Scalar potential With $P \times CP \times Z_2$ and 2 Bi-doublets

We write down the Higgs potential in terms of the $P \times CP \times Z_2$ invariant part V_{Inv} , and soft CP and Z_2 breaking part V_{soft} that conserves P . We note from equations (3) and (2) that

$$V_{Higgs} = V_{Inv.} + V_{soft}$$

with

$$V_{Inv.} = V_{tri}(\Delta_L, \Delta_R) + V_{bi}(\phi_s, \phi_a) + \sum_{z=a,s} V(\phi_z, \Delta_L, \Delta_R)$$

where $V_{tri}(\Delta_L, \Delta_R) + V(\phi_z, \Delta_L, \Delta_R)$ is the most general potential with 1 bi-doublet ϕ_z as given in reference [14].

$$V_{tri}(\Delta_L, \Delta_R) = \{-\mu_3^2 Tr(\Delta_R \Delta_R^\dagger) + \rho_1 Tr[(\Delta_R \Delta_R^\dagger)]^2 + \rho_2 Tr(\Delta_R \Delta_R) Tr(\Delta_R^\dagger \Delta_R^\dagger) + \rho_4 Tr(\Delta_L \Delta_L) Tr(\Delta_R^\dagger \Delta_R^\dagger) + R \leftrightarrow L\} + \rho_3 Tr(\Delta_L \Delta_L^\dagger) Tr(\Delta_R \Delta_R^\dagger) \quad (A1)$$

$$V(\phi_z, \Delta_L, \Delta_R) = -\mu_{1z}^2 Tr(\phi_z^\dagger \phi_z) - \mu_{2z}^2 [Tr(\tilde{\phi}_z^\dagger \phi_z) + h.c.] + \lambda_{1z} [Tr(\phi_z^\dagger \phi_z)]^2 + \lambda_{3z} Tr(\tilde{\phi}_z \phi_z^\dagger) Tr(\tilde{\phi}_z^\dagger \phi_z) + \{\lambda_{2z} [Tr(\tilde{\phi}_z \phi_z^\dagger)]^2 + \lambda_{4z} Tr(\phi_z \phi_z^\dagger) Tr(\tilde{\phi}_z \phi_z^\dagger) + \beta_{1z} Tr(\phi_z \Delta_R \phi_z^\dagger \Delta_L^\dagger) + \beta_{2z} Tr(\tilde{\phi}_z \Delta_R \phi_z^\dagger \Delta_L^\dagger) + \beta_{3z} Tr(\phi_z \Delta_R \tilde{\phi}_z^\dagger \Delta_L^\dagger) + h.c.\} + \{\alpha_{1z} Tr(\phi_z \phi_z^\dagger) Tr(\Delta_R \Delta_R^\dagger) + R \rightarrow L\} + \{\alpha_{2z} Tr(\phi_z \tilde{\phi}_z^\dagger) Tr(\Delta_R \Delta_R^\dagger) + R \rightarrow L + h.c.\} + \alpha_{3z} [Tr(\phi_z^\dagger \phi_z \Delta_R \Delta_R^\dagger) + Tr(\phi_z \phi_z^\dagger \Delta_L \Delta_L^\dagger)] \quad (A2)$$

$$V_{bi}(\phi_a, \phi_s) = \lambda_{1as} Tr(\phi_a^\dagger \phi_a) Tr(\phi_s^\dagger \phi_s) + \lambda_{2as} Tr(\phi_a \phi_a^\dagger \phi_s \phi_s^\dagger) + \lambda_{3as} Tr(\phi_s \tilde{\phi}_a^\dagger) Tr(\phi_s \tilde{\phi}_a^\dagger) + \dots \quad (A3)$$

$$V_{soft} = -\mu_{1as}^2 Tr(\phi_a^\dagger \phi_s) - \mu_{2as}^2 Tr(\tilde{\phi}_a^\dagger \phi_s) - \mu_{2sa}^2 Tr(\tilde{\phi}_s^\dagger \phi_a) + h.c. \quad (A4)$$

In the above we have used curly brackets so that terms that can be obtained by replacing R by L or those that can be obtained by Hermitian conjugation need not be independently written. Within each curly bracket the op-

erations such as $R \rightarrow L$ or $h.c.$ if mentioned would apply to every term and they generate the remaining terms.

Note also that all the parameters in the above potential are real. α_{2a}, α_{2s} are real due to CP , and the

parameters in V_{soft} are real due to P . The remaining parameters are all real due to P and they are also real due to CP . Either of these symmetries can be used to make the remaining parameters real. Z_2 ensures there are no other terms whose parameters could be non-real. Together these symmetries ensure that all parameters of the Higgs potential are real.

V_{bi} is made of several terms that necessarily involve

both ϕ_a and ϕ_s . Z_2 and gauge symmetry implies that each bi-doublet will occur twice in V_{bi} . In the above we have only written some of the terms of V_{bi} .

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